



Lecture9 (optics) single-mode squeezing

- Single-mode squeezing
- Wigner function
- two-mode squeezing
- GRA experiment

Spontaneous parametric down-conversion
vacuum field (vacuum field) seed

$$\frac{\partial^2 E_1^{(1)}}{\partial t^2} = -\omega_1 E_1^{(1)} - \frac{\chi^{(3)}}{n^2} \frac{\partial^2 (E_2^{(1)} E_p)}{\partial t^2}$$
$$\frac{\partial^2 E_2^{(1)}}{\partial t^2} = -\omega_2 E_2^{(1)} - \frac{\chi^{(3)}}{n^2} \frac{\partial^2 (E_1^{(1)} E_p)}{\partial t^2}$$

(1)

(2)

$$\hat{H} = \hbar \omega_0 (\hat{a}_0^\dagger \hat{a}_0 + \frac{1}{2}) + \hbar \omega_1 (\hat{a}_1^\dagger \hat{a}_1 + \frac{1}{2}) + \hbar \omega_2 (\hat{a}_2^\dagger \hat{a}_2 + \frac{1}{2}) + (-\vec{d} \cdot \vec{E})$$

unperturbed

$$\hat{H}_{int} = -e \chi^{(3)} \left(\frac{E_p}{2} \right)^2 \left(\frac{E_p}{2} \right) (\hat{a}_0^\dagger \hat{a}_0 + \hat{a}_1^\dagger \hat{a}_1)$$

resonant

$$-e \chi^{(3)} \vec{E} \vec{E} \vec{E} \rightarrow -e \chi^{(3)} \vec{E}_0 \vec{E}_0 \vec{E}_p = -e \chi^{(3)} \left(\frac{E_0}{2} \right)^2 \frac{E_p}{2} (\hat{a}_0^\dagger + \hat{a}_0)^2$$

interaction picture

$$\hat{a} \rightarrow \hat{a} e^{-i\omega t}$$
$$\hat{a}^\dagger \rightarrow \hat{a}^\dagger e^{i\omega t}$$
$$\vec{E} \rightarrow \vec{E}_0 \vec{E} = \vec{E}_0 \sqrt{2} \left(\frac{\hat{a} + \hat{a}^\dagger}{2} \right)$$
$$\vec{B} \rightarrow \vec{B}_0 \vec{B} = \vec{E}_0 \sqrt{2} \left(\frac{\hat{a} - \hat{a}^\dagger}{2i} \right)$$
$$\frac{\partial \hat{H}}{\partial t} = \left(\frac{\partial \hat{H}}{\partial t} + \hat{H}_z \right) | \psi \rangle$$
$$\left(\hat{a}_0^\dagger \hat{a}_0 + \hat{a}_1^\dagger \hat{a}_1 \right) | \psi_{vac} \rangle = 0$$

(3)

$$\begin{cases} \pm i \hbar \omega \\ e^{-i \hbar \omega - i \omega t} \rightarrow 1 \\ e^{-i \hbar \omega + i \omega t} \rightarrow \pm i \hbar \omega \end{cases}$$




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Parametric approximation (quantum optics) (3)

→ assume the pump field to be a coherent state, which remains "undepleted" after the interaction

$$\hat{a}|\gamma\rangle = \gamma|\gamma\rangle$$

$$\hat{a}^\dagger|\gamma\rangle = \gamma^*|\gamma\rangle$$

$$\hat{a}^\dagger \hat{a} = \gamma \gamma^*$$

$$i\hbar \frac{\partial \hat{\psi}}{\partial t} = \hat{H} \hat{\psi}, \quad \hat{\psi}(x) = e^{-i\hat{H}t/\hbar} |\psi(0)\rangle$$

Squeezed state: $|\xi\rangle = \frac{1}{\sqrt{2}} e^{\frac{1}{2}(\hat{a}^\dagger - \hat{a})^2} |0\rangle$

Coherent state: $|\alpha\rangle = e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}} |0\rangle$

Squeezing operator: $\hat{S}(\xi) = \frac{1}{2} \hat{H} \hat{S}^\dagger(\xi) \hat{H}$, $\xi = \eta e^{i\theta}$

Displacement operator: $\hat{D}(\alpha) = e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}$, $\alpha = \mu e^{i\phi}$

$|\psi\rangle = |0\rangle|\gamma\rangle \rightarrow \hat{H}_{int} = i\hbar(\hat{a}_0^\dagger \hat{a}^\dagger - \hat{a} \hat{a}^\dagger) \frac{1}{2} \rightarrow \text{nonlinear}$

$\hat{H}_{int} = i\hbar(\hat{a}_p^\dagger - \hat{a}_p) \rightarrow \text{linear}$

④

$$|\psi(t)\rangle = \hat{S}(\xi)|0\rangle = e^{\frac{1}{2}(\hat{a}^\dagger \xi - \hat{a} \xi^*)}|0\rangle = \sum_m C_m |2m\rangle$$

$$\hat{e}^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\hat{e}^{\hat{a}} = 1 + \hat{a} + \frac{\hat{a}^2}{2!} + \frac{\hat{a}^3}{3!} + \dots$$

$$\hat{1}|0\rangle = |0\rangle$$

$$+ \left[\frac{1}{2}(\hat{a}^\dagger \xi - \hat{a} \xi^*) \right] |0\rangle = |2\rangle$$

$$+ \frac{1}{2!} \left[\frac{1}{2}(\hat{a}^\dagger \xi - \hat{a} \xi^*) \right]^2 |0\rangle = |4\rangle$$

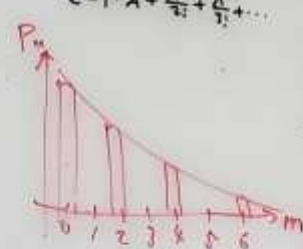
$$+ \frac{1}{3!} \left[\frac{1}{2}(\hat{a}^\dagger \xi - \hat{a} \xi^*) \right]^3 |0\rangle = |6\rangle$$

$$10\rangle$$

$$12\rangle$$

$$14\rangle$$

$$16\rangle$$



$$P_{2m} = |C_m|^2 = \frac{(2m)!}{2^{2m} (m!)^2} \frac{(\tanh(r))^{2m}}{\cosh(r)}, \quad \xi = r e^{i\theta}$$

$$P_{2m+1} = 0$$





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